Review of Differential Equations

1. $\frac{dy}{dx} = e^{x-y} = e^x e^{-y}$

$\Rightarrow e^y dy = e^x dx \Rightarrow \int e^y dy = \int e^x dx$

$\Rightarrow e^y = e^x + C$

$\therefore y = \ln(e^x + C)$

2. $x^2(y^2+1)dx + y\sqrt{x^3+1}\,dy = 0$

$\Rightarrow \frac{x^2}{\sqrt{x^3+1}}dx = \frac{-y}{y^2+1}dy$

let $u = x^3+1 \rightarrow du = 3x^2dx$

let $w = y^2+1 \rightarrow dw = 2y\,dy$

$\Rightarrow \frac{1}{3}\int u^{-1/2}\,du = -\frac{1}{2}\int w^{-1}\,dw$

$\Rightarrow \frac{1}{3}\cdot\frac{2}{1}u^{1/2} + C_1 = -\frac{1}{2}\ln|w|$

$\Rightarrow \frac{2}{3}\sqrt{x^3+1} + C = -\frac{1}{2}\ln|y^2+1|$

$\Rightarrow -\frac{4}{3}\sqrt{x^3+1} + C = \ln|y^2+1|$ exponentiating $\Rightarrow Ae^{-\frac{4}{3}\sqrt{x^3+1}} = |y^2+1|$

$\Rightarrow y^2 = -1 + Ae^{-\frac{4}{3}\sqrt{x^3+1}}$ $\therefore y = \pm\sqrt{-1+Ae^{-\frac{4}{3}\sqrt{x^3+1}}}$ (wow!)

3. $\sqrt{2xy}\frac{dy}{dx} = 1 \Rightarrow \sqrt{y}\,dy = \frac{1}{\sqrt{2x}}dx$

$\Rightarrow \int y^{1/2}dy = \frac{1}{\sqrt{2}}\int x^{-1/2}dx \Rightarrow \frac{2}{3}y^{3/2} = \frac{1}{\sqrt{2}}\cdot\frac{2}{1}x^{1/2} + C = \sqrt{2x} + C$

$\Rightarrow y^{3/2} = \frac{3}{2}\sqrt{2x} + C$ $\therefore y = \left(\frac{3}{2}\sqrt{2x} + C\right)^{2/3}$

4. $\ln x\frac{dx}{dy} = \frac{x}{y} \Rightarrow \frac{1}{x}\ln x\,dx = \frac{1}{y}dy \Rightarrow \int\frac{1}{x}\ln x\,dx = \int\frac{1}{y}dy$ $u = \ln x$, $du = \frac{1}{x}dx$

$\Rightarrow \int u\,du = \int\frac{1}{y}dy \Rightarrow \frac{u^2}{2} + C_1 = \ln|y|$

$\Rightarrow \frac{1}{2}(\ln x)^2 + C = \ln|y| \Rightarrow |y| = Ae^{\frac{1}{2}(\ln x)^2}$

$\therefore y = \pm Ae^{\frac{1}{2}(\ln x)^2}$

5. $(x+1)\frac{dy}{dx} = x(y^2+1)$

$$\Rightarrow \int \frac{1}{y^2+1}\,dy = \int \frac{x}{x+1}\,dx = \int \frac{x+1}{x+1}\,dx - \int \frac{1}{x+1}\,dx$$

$$\tan^{-1} y = x + \ln|x+1| + C$$

6. $x^2 y \frac{dy}{dx} = (x+1)\csc y$

$$\Rightarrow \frac{y}{\csc y}\,dy = \frac{x+1}{x^2}\,dx$$

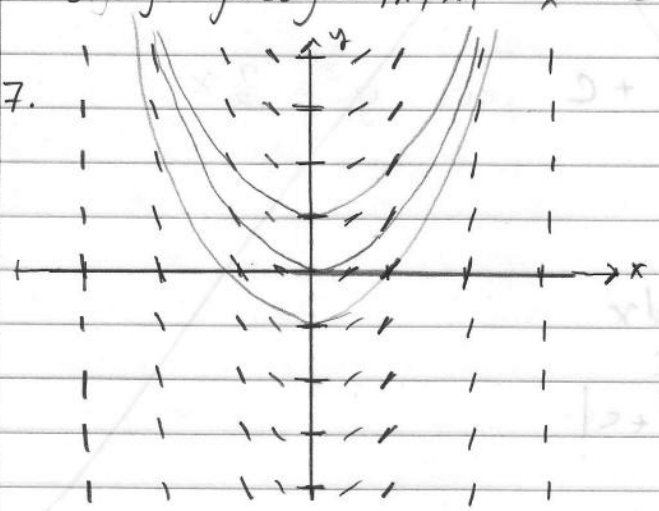
$$\int y \sin y\,dy = \int \frac{1}{x}\,dx + \int \frac{1}{x^2}\,dx$$

$\Rightarrow$ $u = y \quad dv = \sin y\,dy$
$du = dy \quad v = -\cos y$

$$-y\cos y + \int \cos y\,dy = \ln|x| - \frac{1}{x} + C$$

$$\sin y - y\cos y = \ln|x| - \frac{1}{x} + C$$

7.

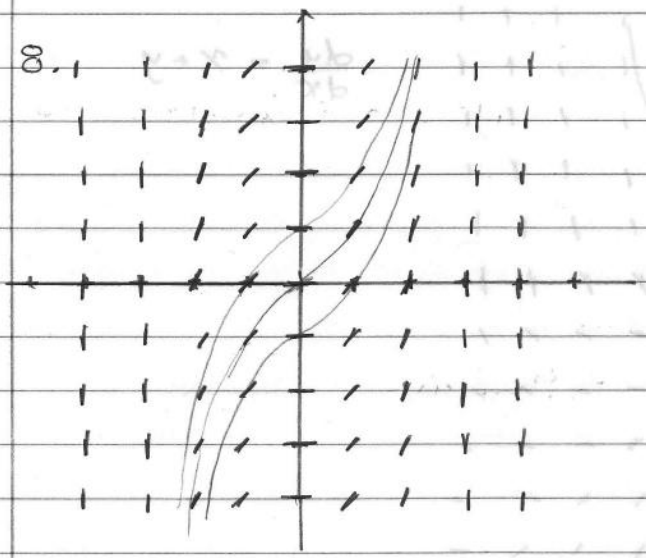


$\frac{dy}{dx} = 2x$

$dy = 2x\,dx$

$y = x^2 + C$

8.

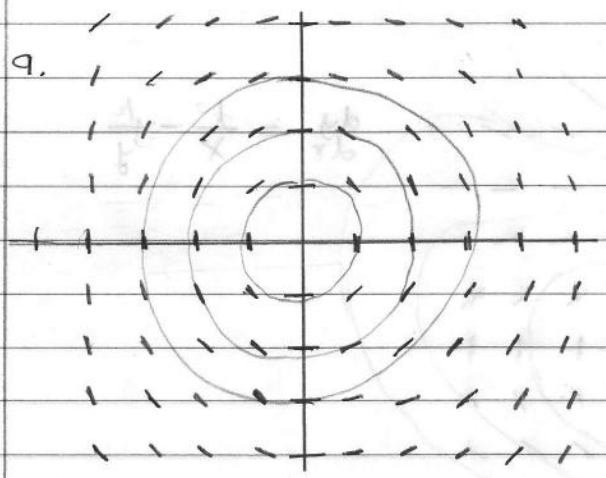


$\frac{dy}{dx} = x^2$

$dy = x^2 dx$

$y = \frac{1}{3}x^3 + C$

9.



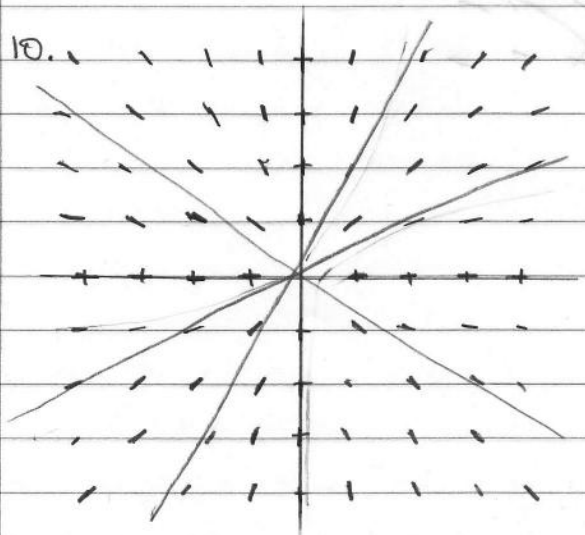
$\frac{dy}{dx} = -\frac{x}{y}$

$y\,dy = -x\,dx$

$\frac{1}{2}y^2 = -\frac{1}{2}x^2 + C$

$x^2 + y^2 = C$

10.



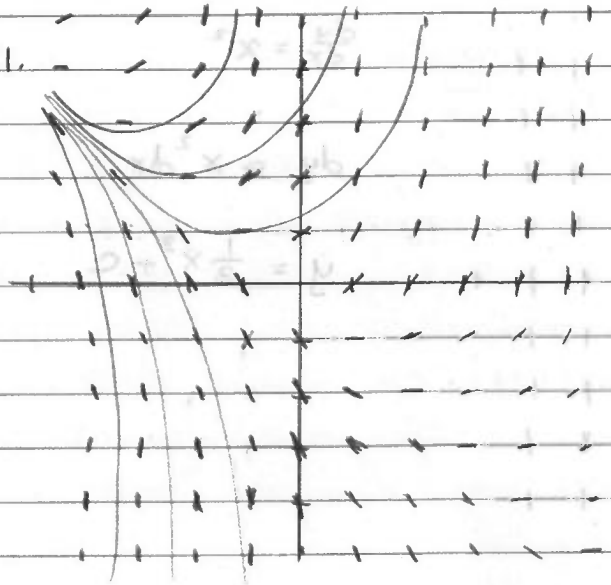
$\frac{dy}{dx} = \frac{y}{x}$

$\frac{1}{y}dy = \frac{1}{x}dx$

$\ln y = \ln x + c$

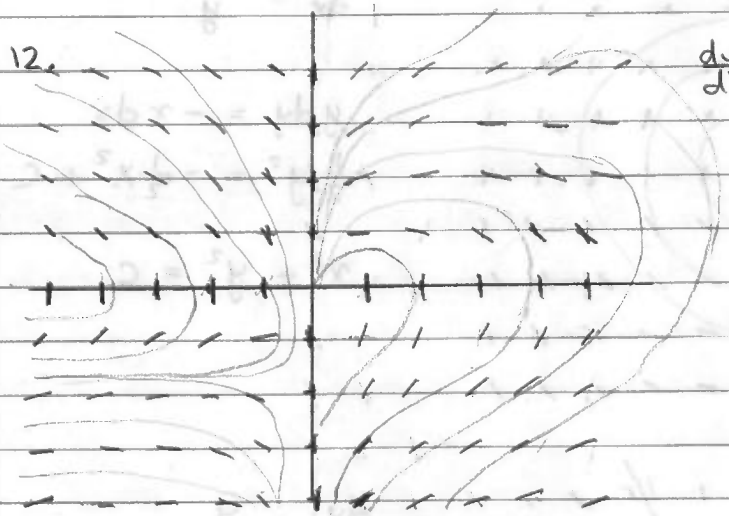
$\Rightarrow y = cx$

11.



$$\frac{dy}{dx} = x + y$$

12.



$$\frac{dy}{dx} = \frac{1}{x} - \frac{1}{y}$$

13.

x	y	Δx	$\frac{dy}{dx} = x+y$	$\Delta y = \frac{dy}{dx}\Delta x$
0	1	.5	$0+1=1$	$1\cdot(.5)=.5$
0.5	1.5	.5	$0.5+1.5=2$	$2\cdot(.5)=1$
1.0	2.5	.5	$1+2.5=3.5$	$3.5(.5)=1.75$
1.5	4.25	.5	$1.5+4.25=5.75$	$5.75(.5)=2.875$
2	7.125			

14.

x	y	Δx	$\frac{dy}{dx} = \frac{1}{x} - \frac{1}{y}$	$\Delta y = \frac{dy}{dx}\Delta x$
1	1	.25	$\frac{1}{1} - \frac{1}{1} = 0$	$0(.25)=0$
1.25	1	.25	$\frac{1}{1.25} - \frac{1}{1} = -.2$	$-.2(.25)=-.05$
1.5	.95	.25	$\frac{1}{1.5} - \frac{1}{.95} = -.386$	$-.386(.25)=-.096$
1.75	.854	.25	$\frac{1}{1.75} - \frac{1}{.854} = -.599$	$-.599(.25)=-.150$
2	.704			