Test 4 A d

1a) $\int_0^3 f(x)\,dx = -2$

b) $\int_3^6 f(x)\,dx = 4$

c) $\int_0^3 f(x)\,dx = -2$

d) $f_{avg} = \frac{1}{6-0}\int_0^6 f(x)\,dx = \frac{1}{6}\left[-\int_0^3 f(x)\,dx + \int_3^6 f(x)\,dx\right] = \frac{1}{6}\left[-2+4\right] = \frac{1}{3}$

2a) $\int_{-6}^{-6} f(x)\,dx = 0$

b) $\int_{-6}^{-2} f(x)\,dx = 8$ because, f is even $\Rightarrow \int_{-6}^{-2} f(x)\,dx = \int_2^6 f(x)\,dx$

c) $\int_0^6 f(x)\,dx = \int_0^2 f(x)\,dx + \int_2^6 f(x)\,dx = -3 + 8 = 5$

d) $\int_6^2 f(x)\,dx = -\int_2^6 f(x)\,dx = -8$

3a) Donations arrived at the greatest rate on Jan 1, $t = 0$ weeks.

b) Left: $\int_0^{10} R(t)\,dt \approx (5 + 4.5 + 2.5 + 0.6 + 0.1)(2) = 25.4$

Right: $\int_0^{10} R(t)\,dt \approx (4.5 + 2.5 + 0.6 + 0.1 + 0)(2) = 15.4$

Average estimate is $\frac{25.4 + 15.4}{2} = \$20.4$ thousand or $\$20,400$

c) $R_{avg} = \frac{1}{10-0}\int_0^{10} R(t)\,dt \approx \frac{20.4}{10} = \$2,040$ /week

d) unc $= |\text{first} - \text{last}|\,\Delta t = |5000 - 0|(2) = \$10,000$

e) $1000 = |5000 - 0|\,\Delta t \Rightarrow \Delta t = \frac{1000}{5000} = .2$ week or 1.4 days

4a) $\int (\sin x + 4x^3)\,dx = -\cos x + x^4 + C$

b) $\int (\sec^2 y + 6)\,dy = \tan y + 6y + C$

c) $\int \frac{x^6 - 1}{x^5}\,dx = \int (x - x^{-5})\,dx = \frac{1}{2}x^2 + \frac{1}{4}x^{-4} + C$

d) $\int \left(\sqrt[3]{x} - \frac{1}{\sqrt[3]{x}}\right)dx = \int (x^{1/3} - x^{-1/3})\,dx = \frac{3}{4}x^{4/3} - \frac{3}{2}x^{2/3} + C$

5a $\int_0^4 (x+2)\,dx = \left[\frac{1}{2}x^2+2x\right]_0^4 = \left[\frac{1}{2}(4)^2+2(4)\right]-\left[\frac{1}{2}(0)^2+2(0)\right] = 16$

b) $\int_1^4 \frac{1}{x^2}\,dx = \int_1^4 x^{-2}\,dx = \left[-x^{-1}\right]_1^4 = \left[-\frac{1}{4}\right]-\left[-\frac{1}{1}\right] = \frac{3}{4}$

c) $\int_0^4 r(4-r)\,dr = \int_0^4 (4r-r^2)\,dr = \left[2r^2-\frac{1}{3}r^3\right]_0^4$

$= \left[2(4)^2-\frac{1}{3}(4)^3\right]-\left[2(0)^2-\frac{1}{3}(0)^3\right] = \frac{32}{3}$

6. $\frac{dy}{dx} = 2x-3 \Rightarrow y = \int(2x-3)\,dx = x^2-3x+C$

using $(2,3)$: $3 = (2)^2-3(2)+C \Rightarrow C = 5$

$\therefore\ y = x^2-3x+5$

<u>BONUS</u>: According to the FTC, $\int_0^x f(t)\,dt = F(x)-F(0)$

Then $\frac{d}{dx}\int_0^x f(t)\,dt = \frac{d}{dx}\left[F(x)-F(0)\right]$

$= f(x) - 0$ since $F' = f$ and $F(0)$ is constant.

$= f(x)$