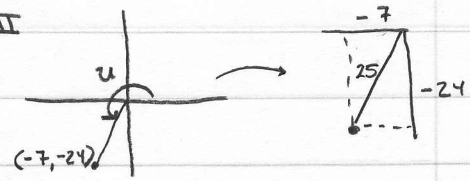
Review 5

1a) $\tan u = 24/7$, $\sin u < 0 \Rightarrow u$ in QIII

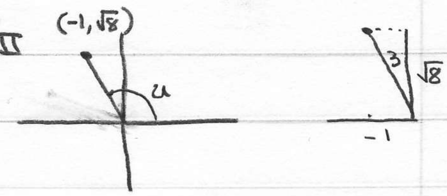


$\sin u = -24/25$ $\quad\quad$ $\csc u = -25/24$

$\cos u = -7/25$ $\quad\quad$ $\sec u = -25/7$

$\tan u = 24/7$ $\quad\quad$ $\cot u = 7/24$

b) $\sec u = -3$, $\pi/2 < u < \pi \Rightarrow u$ in QII



$\sin u = \sqrt{8}/3$ $\quad\quad$ $\csc u = 3/\sqrt{8}$

$\cos u = -1/3$ $\quad\quad$ $\sec u = -3$

$\tan u = -\sqrt{8}$ $\quad\quad$ $\cot u = -1/\sqrt{8}$

2a) $\cot x \cdot \tan x = 1$

b) $$\frac{\sin^2 x}{1-\cos x} = \frac{1-\cos^2 x}{1-\cos x} = \frac{(1-\cos x)(1+\cos x)}{(1-\cos x)} = 1+\cos x.$$

c) $$\frac{1+\sin\theta}{\cos\theta} + \frac{\cos\theta}{1+\sin\theta} = \frac{(1+\sin\theta)^2+\cos^2\theta}{\cos\theta(1+\sin\theta)} = \frac{1+2\sin\theta+\sin^2\theta+\cos^2\theta}{\cos\theta(1+\sin\theta)}$$

$$= \frac{2+2\sin\theta}{\cos\theta(1+\sin\theta)} = \frac{2(1+\sin\theta)}{\cos\theta(1+\sin\theta)} = \frac{2}{\cos\theta} \text{ or } 2\sec\theta$$

3a) $\cos\alpha \csc\alpha = \cot\alpha$

$$\cos\alpha \cdot \frac{1}{\sin\alpha} = \cot\alpha$$

$\cot\alpha = \cot\alpha$ QED.

b) $\sin(\pi-\theta) - \cos\left(\frac{\pi}{2}+\theta\right) = 2\sin\theta$

$\sin\pi\cdot\cos\theta - \cos\pi\cdot\sin\theta - \cos\frac{\pi}{2}\cos\theta + \sin\frac{\pi}{2}\sin\theta = 2\sin\theta$

$0\cdot\cos\theta - (-1)\sin\theta - 0\cdot\cos\theta + (1)\sin\theta = 2\sin\theta$

$2\sin\theta = 2\sin\theta$ QED.

c) $(\csc x + 1)(\csc x - 1) = \cot^2 x$

$\csc^2 x - 1 = \cot^2 x$

$\cot^2 x = \cot^2 x$ QED

d) $\sin 3x = 3\sin x - 4\sin^3 x$ $\quad\quad$ $(\sin 3x = \sin(2x+x))$

$\sin 2x \cos x + \cos 2x \sin x = 3\sin x - 4\sin^3 x$

$2\sin x\cdot\cos^2 x + \cos^2 x\cdot\sin x - \sin^2 x\cdot\sin x = 3\sin x - 4\sin^3 x$

3d cont. $2\sin x(1-\sin^2 x) + (1-\sin^2 x)\sin x - \sin^3 x = 3\sin x - 4\sin^3 x$

$2\sin x - 2\sin^3 x + \sin x - \sin^3 x - \sin^3 x = 3\sin x - 4\sin^3 x$

$3\sin x - 4\sin^3 x = 3\sin x - 4\sin^3 x$ QED.

4a) $2\cos x = -1 \Rightarrow \cos x = -\frac{1}{2} \Rightarrow x = \frac{2\pi}{3} + 2\pi k,\ x = \frac{4\pi}{3} + 2\pi k$ on $(-\infty, \infty)$

b) $4\sin^2 x = 3 \Rightarrow \sin^2 x = \frac{3}{4} \Rightarrow \sin x = \pm\frac{\sqrt{3}}{2} \Rightarrow x = \frac{\pi}{3}, \frac{2\pi}{3}$ on $[0, \pi]$

c) $\tan^2 u - 6\tan u + 4 = 0 \Rightarrow v^2 - 6v + 4 = 0 \Rightarrow v = \frac{6 \pm \sqrt{20}}{2} = 3 \pm \sqrt{5} \approx 5.236, 0.764$

$\therefore u = \tan^{-1}(5.236) = 1.382,\ 1.382 + \pi = 4.524$ on $[0, 2\pi)$

$u = \tan^{-1}(0.764) = 0.652,\ 0.652 + \pi = 3.794$

d) $\sin 2x = \cos x \Rightarrow 2\sin x\cos x - \cos x = 0 \Rightarrow \cos x(2\sin x - 1) = 0$

$\therefore \cos x = 0 \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$ on $[0, 2\pi)$.

$\sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}$

5. $\tan 15^\circ = \tan(60^\circ - 45^\circ) = \frac{\tan 60^\circ - \tan 45^\circ}{1 + \tan 60^\circ \tan 45^\circ} = \frac{\sqrt{3} - 1}{1 + \sqrt{3}}$

6. $\cos\frac{13\pi}{12} = \cos\left(\frac{5\pi}{6} + \frac{\pi}{4}\right) = \cos\frac{5\pi}{6}\cos\frac{\pi}{4} - \sin\frac{\pi}{6}\sin\frac{\pi}{4} = -\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} - \frac{1}{2}\cdot\frac{\sqrt{2}}{2} = \frac{-\sqrt{6} - \sqrt{2}}{4}$

7. $\tan\frac{5\pi}{12} = \tan\left(\frac{5\pi/6}{2}\right) = \frac{\sin 5\pi/6}{1 + \cos 5\pi/6} = \frac{\frac{1}{2}}{1 - \sqrt{3}/2} = \frac{1}{2 - \sqrt{3}}$ or $\frac{2 + \sqrt{3}}{7}$

8. $\sin 22.5^\circ = \sin\left(\frac{45^\circ}{2}\right) = \pm\sqrt{\frac{1 - \cos(45)}{2}} = \pm\sqrt{\frac{1 - 1/\sqrt{2}}{2}} = \pm\sqrt{\frac{2 - \sqrt{2}}{4}} = \frac{\sqrt{2 - \sqrt{2}}}{2}$

9. $\cos 2x = 2\cos^2 x - 1$ If $\cos x = 0.6$, then $\cos 2x = 2(0.6)^2 - 1 = -0.28$

10. From the diagram, $\sin\theta = \frac{1}{\sqrt{5}}$, $\cos\theta = \frac{2}{\sqrt{5}}$, $\tan\theta = \frac{1}{2}$. Then,

$\sin 2\theta = 2\sin\theta\cos\theta = 2\left(\frac{1}{\sqrt{5}}\right)\left(\frac{2}{\sqrt{5}}\right) = \frac{4}{5}$ $\cos 2\theta = \cos^2\theta - \sin^2\theta = \left(\frac{2}{\sqrt{5}}\right)^2 - \left(\frac{1}{\sqrt{5}}\right)^2 = \frac{3}{5}$ $\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{4}{3}$

11. From the diagram, $\sin\varphi = \frac{3}{\sqrt{10}}$, $\cos\varphi = \frac{1}{\sqrt{10}}$, $\tan\varphi = 3$. Then,

$\sin(\varphi - \theta) = \sin\varphi\cos\theta - \cos\varphi\sin\theta = \frac{3}{\sqrt{10}}\cdot\frac{2}{\sqrt{5}} - \frac{1}{\sqrt{10}}\cdot\frac{1}{\sqrt{5}} = \frac{5}{\sqrt{50}}$

$\cos(\theta + \varphi) = \cos\theta\cos\varphi - \sin\theta\sin\varphi = \frac{2}{\sqrt{5}}\cdot\frac{1}{\sqrt{10}} - \frac{1}{\sqrt{5}}\cdot\frac{3}{\sqrt{10}} = \frac{-1}{\sqrt{50}}$

$\tan(\theta - \varphi) = \frac{\tan\theta - \tan\varphi}{1 + \tan\theta\tan\varphi} = \frac{\frac{1}{2} - 3}{1 + \frac{1}{2}\cdot 3} = -1$

12. $\cos x$ and x represent different families of function. Therefore, algebra which solves by using inverses cannot solve the equation. Using TI-83: $x \approx .739$

13. $\frac{\sin(x+y) + \sin(x-y)}{2} = \frac{\sin x\cos y + \cos x\sin y + \sin x\cos y - \cos x\sin y}{2}$

$= \frac{2\sin x\cos y}{2} = \sin x\cos y$ QED.