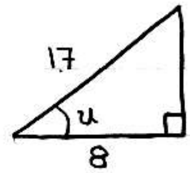
Test 5c

1. $\sin u = 15/17$ $\quad \csc u = 17/15$

$\cos u = 8/17$ $\quad \sec u = 17/8$

$\tan u = 15/8$ $\quad \cot u = 8/15$



$15^2 + 8^2 = 289$

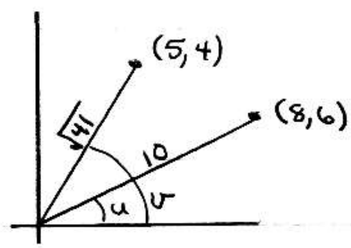
$\sqrt{289} = 17$

2. $\tan(105°) = \tan(60° + 45°) = \dfrac{\tan 60 + \tan 45}{1 - \tan 60 \cdot \tan 45} = \boxed{\dfrac{\sqrt{3}+1}{1-\sqrt{3}}}$

3. $\cos x = 0.8 \Rightarrow \sin x = \sqrt{1-(0.8)^2} = 0.6 \therefore \sin 2x = 2\sin x \cdot \cos x = 2(0.8)(0.6) = \boxed{0.96}$

4. $\sin(u-v) = \sin u \cdot \cos v - \cos u \cdot \sin v$

$= \frac{6}{10} \cdot \frac{5}{\sqrt{41}} - \frac{8}{10} \cdot \frac{4}{\sqrt{41}} = \boxed{\frac{-1}{5\sqrt{41}}}$



$\cos(u+v) = \cos u \cdot \cos v - \sin u \cdot \sin v$

$= \frac{8}{10} \cdot \frac{5}{\sqrt{41}} - \frac{6}{10} \frac{4}{\sqrt{41}} = \boxed{\frac{8}{5\sqrt{41}}}$

$\tan(2v) = \dfrac{2\tan v}{1-\tan^2 v} = \dfrac{2\left(\frac{4}{5}\right)}{1-\left(\frac{4}{5}\right)^2} = \boxed{\dfrac{40}{9}}$

5a) $\cot x \cdot \sec x - \csc x = \dfrac{\cos x}{\sin x} \cdot \dfrac{1}{\cos x} - \dfrac{1}{\sin x} = \dfrac{1}{\sin x} - \dfrac{1}{\sin x} = \boxed{0}$

b) $\cos 2x + \sin^2 x = \cos^2 x - \sin^2 x + \sin^2 x = \boxed{\cos^2 x}$

c) $\dfrac{\sec x \cdot \csc x}{2} = \dfrac{1}{2\sin x \cos x} = \boxed{\dfrac{1}{\sin 2x} \text{ or } \csc 2x}$

6a) $\tan^2 x + 1 = \dfrac{\sin^2 x}{\cos^2 x} + \dfrac{\cos^2 x}{\cos^2 x} = \dfrac{\sin^2 x + \cos^2 x}{\cos^2 x} = \dfrac{1}{\cos^2 x} = \sec^2 x$ QED!

b) $\sqrt{\dfrac{1-\cos\phi}{1+\cos\phi}} = \sqrt{\dfrac{1-\cos\phi}{1+\cos\phi}} \cdot \sqrt{\dfrac{1-\cos\phi}{1-\cos\phi}} = \dfrac{1-\cos\phi}{\sqrt{1-\cos^2\phi}} = \dfrac{1-\cos\phi}{\sqrt{\sin^2\phi}} = \dfrac{1-\cos\phi}{|\sin\phi|}$ QED!

7a) $2\sin x = \sqrt{3} \Rightarrow \sin x = \dfrac{\sqrt{3}}{2} \Rightarrow$ $\boxed{\begin{array}{l} x = \frac{\pi}{3} + 2\pi n \\ x = \frac{2\pi}{3} + 2\pi n \end{array}}$

on $(-\infty, \infty)$

7b) $\tan 2x = 1 \Rightarrow 2x = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4} \Rightarrow \boxed{x = \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}}$

$x \in [0, 2\pi) \Rightarrow 2x \in [0, 4\pi)$

7c) $\cos^2 u - 2\cos u - 3 = 0 \Rightarrow (\cos u - 3)(\cos u + 1) = 0$ $\quad u \in [0, 2\pi)$

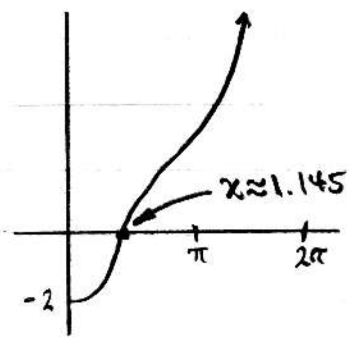
$\Rightarrow \cos u = 3$ (impossible!), $\cos u = -1$ $\quad \therefore \boxed{u = \pi}$

7d) $4\cos^2 x = 3 \Rightarrow \cos^2 x = \frac{3}{4} \Rightarrow \cos x = \pm \frac{\sqrt{3}}{2} \Rightarrow \boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}}$

$x \in [0, \pi)$ $\quad$ ($\frac{7\pi}{6}$ and $\frac{11\pi}{6}$ are not in the domain).

8. $\sin^4 x + x^2 = 2 \Rightarrow \sin^4 x + x^2 - 2 = 0$

$\Rightarrow \boxed{x = 1.145}$



<u>BONUS</u>: $\cos(x) + \cos(y) = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$

The product to sum formula is $\cos u \cos v = \frac{1}{2}(\cos(u+v) + \cos(u-v))$

So, if $u = \frac{x+y}{2}$ and $v = \frac{x-y}{2}$ then $u+v = x$ and $u-v = y$

$\therefore \cos(x) + \cos(y) = 2 \cdot \frac{1}{2}(\cos(x) + \cos(y))$

$= \cos(x) + \cos(y)$ QED!